EXHIBIT R

TO RULE 4.2 STATEMENT OF DR. DOUGHERTY

FRINGE CAPACITANCE AND SCATTERING FIELD OF A FERROELECTRIC PLANAR CAPACITOR

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In the experimental investigation and use at microwave frequencies of thin ferroelectric films deposited on a dielectric substrate [1], a planar arrangement of the electrodes is usually employed. Because of the small width of the gap between the planar electrodes (a few micrometers) the length of the gap is usually less than the dimensions of the film, so that there is a fringing capacitance due to the electric scattering field in the part of the film free from the electrodes (the flanges). The technological possibilities of reducing the flanges are limited, so that it is important to estimate the contribution of the fringing capacitance to the over-all capacitance of the planar capacitor.

Because of the high value of the permittivity of a ferroelectric film ($\varepsilon \sim 10^3$), the electric scattering field in the medium surrounding the capacitor can be neglected, assuming that the normal component of the field at the boundary of the ferroelectric is zero. It should be noted that the boundary value problem with the above boundary conditions (the so-called "magnetic wall") has been studied in some detail in the literature [2]. Previously, using the method of conformal transformation, the electric field was determined [3] and also the capacitance per unit length [3,4] of a planar ferroelectric capacitor. In the present paper we consider the electric scattering field and the related fringing capacitance in a ferroelectric film with a planar arrangement of the electrodes.

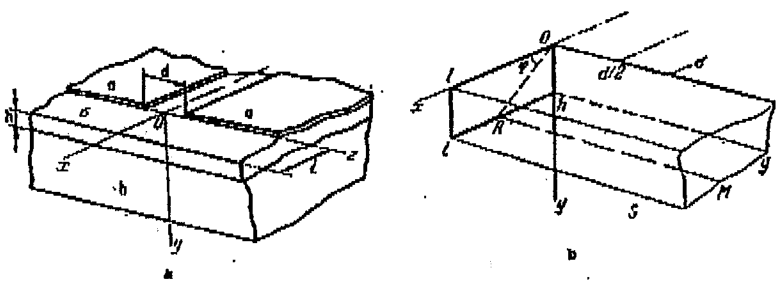

Fig. 1

The arrangement of the planar electrodes with a gap d with respect to the ferroelectric film of thickness h is shown in Fig. 1a, where a are the electrodes, σ is the ferroelectric film, and b is the dielectric substrate. The surface of the film can be regarded as a "magnetic wall" so that the value of the fringing capacitance (for specified d and h) is determined by the dimension of the flange l. The symmetry of the problem [4] enables us to choose the fringing capacitance shown in Fig. 1b as the theoretical model, where the electrodes are represented by the heavy lines, and a is a plane rectangular electrode. In this model there is a difference of potential U/2 between the plane $Olth$ and a semi-infinite filament (σ), situated along the edge of the "magnetic" surface S of the rectangular cross section. The surface S bounds the region in which the electric scattering field E is defined. The dielectric filling of this region is characterized by a permittivity ε, which is assumed to be uniform, isotropic, and independent of U (in a weak field). It is obvious that the fringing capacitance C_f is half the capacitance of the model considered.

The electric field E in principle can be found from Laplace's equation, for example, using the method described in [2]. However, because of the complexity of the boundary conditions the series obtained are quite complex and do not converge sufficiently rapidly

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In the present paper we use a different method, based on the fact that the two-dimensional field E_0 for any plane passing through the axis of the filament (for example, for the planes zORM), is characterized by identical boundary conditions and differs only in the scale along the OR direction. It follows from Fig. 1b that the length $OR=R$ for specified h and l is defined as a function of the angle φ by the relations

$$R = \begin{cases} l/\cos\varphi & \text{for } 0 \leqslant \varphi \leqslant \varphi_0, \\ h/\sin\varphi & \text{»} \ \varphi_0 \leqslant \varphi \leqslant \pi/2, \end{cases} \tag{1}$$

where $\varphi_0 = \operatorname{arc\,tg}(h/l)$.

On the other hand, the field E_0 for the plane zOhg represents the field in a section of the uniform planar structure from the solution of the two-dimensional problem [3]. Hence, the required field E can be represented in the form of the superposition of two-dimensional fields E_0, differing in scale for different values of the angle φ. According to [3], the modulus of the field strength E_0 for the zORM plane in a system of coordinates (for specified φ) has the form

$$E_0(r,z)\,|_{R=\text{const}} = \frac{\pi U}{2K(k')R}\,[4k^2 \operatorname{sh}^2 \pi z/R - 4k(k^2+1)\operatorname{ch} \pi z/R \cos \pi r/R + 4k^2 \cos^2 \pi r/R + (k^2+1)^2]^{-1/4}. \tag{2}$$

Here K(k') is the complete elliptic interval of the first kind of the supplementary modulus $k' = \sqrt{1-k^2}$; $k = \exp(-\pi d/2R)$. Now the scattering field can be defined by the expression

$$E(r,\varphi,z) = E_0(r,z)\,|_{R=R(\varphi)}. \tag{3}$$

For a specified potential difference, Eqs. (2) and (3) enable the value of the fringing capacitance to be determined in terms of the total charge or the total energy of the scattering field. However, the scale invariance of the problem in planes passing through the axis of the filament enables the capacitance per unit length of the uniform planar structure (the section zOhg) to be used to determine C_k. The capacitance per unit length [3,4] is given by the relation $C_u = \varepsilon K(k_h)/K(k_h')$, where $k_h = \exp(-\pi d/2h)$. Consequently, the elementary capacitance taken over the angular sector φ, $\varphi + d\varphi$, has the form $dC_k = \varepsilon [K(k)/K(k')] R(\varphi) d\varphi$. Expressing $R(\varphi)$ and φ from (1) in terms of k, we finally obtain

$$2C_k = -\pi\varepsilon d \int_{k_l}^{k_0} \frac{K(k)\,dk}{k \ln k K(k') \sqrt{\ln^2 k_0 - \ln^2 k}} - \pi\varepsilon d \int_{k_h}^{k_0} \frac{K(k)\,dk}{k \ln k K(k') \sqrt{\ln^2 k_h - \ln^2 k}}. \tag{4}$$

Here $k_l = \exp(-\pi d/2l)$, $k_0 = \exp(-\pi d/2\sqrt{h^2+l^2})$, and the appearance of the minus sign is due to the negative values of the function ln k, since 0 < k < 1.

We will estimate the contribution of the fringing capacitance to the capacitance of the planar capacitance, which is equal to LC_u, where L is the length of the gap. For this purpose we will represent the relation $\Delta = 2C_k/LC_u$ in the form of a series in powers of $\delta = l/L$. The coefficients for the even powers of the series are zero, since the function $C_k(l)$ is odd. Using a fundamental theory of integral calculus for Eq. (4) and the properties of the elliptic integrals (see for example, [5]), it can be shown that

$$\Delta = 2\delta + 1/3\,(L/h)^2 \left[1 - \frac{\pi \ln k_h}{2(1-k_h^2)K(k_h)K(k_h')}\right]\delta^3 + \dots \tag{5}$$

In cases of practical interest the values of the coefficient of the form $g = h/d = -(\pi/2)\ln k_h$ lie in the range from 0.2 to 5 [3]. For these values of g the contribution from the second term in the square brackets of Eq. (5) is 0.3–0.8. Hence, the departure of the functional dependence $\Delta(\delta)$ from linear is greatest for thin films. For example, for $L = 10^{-3}$ m and $\delta = 0.01$ for a film $5 \cdot 10^{-6}$ m thick, the value of $\Delta = 0.04$, and for $h = 10^{-6}$ m we have $\Delta = 0.4$. Hence, the contribution of the fringing capacitance to the over-all capacitance of the capacitor for films of thickness ≤1 μm is considerable. This must be taken into account in experimental investigations of the voltage-capacitance and voltage-current

characteristics of ferroelectric planar capacitors. The calculation of C_k carried out above enables the fringing capacitance to be eliminated from the results of measurements, and enables the accuracy to be increased when determining the electrical characteristics of film ferroelectric materials.

Like the results in [6], the method described above can be recommended for analyzing wave processes (the field and radiation resistance in a ferroelectric film) for planar microwave structures [1].

It should be noted that in the results obtained we have ignored the so-called dimensional effect [1], in view of the following factors. Firstly, the presence of spatial dispersion imposes certain limitations on the applicability of the method of conformal representation, used previously to obtain planar solutions for E_0 and C_u. Secondly, even if the correctness of this method [7] and the values of E_0 and C_u obtained are proved, the presence of spatial dispersion leads to violation of the scale invariance of the problem [8]. Hence, it is not obvious that additional renormalization of the solutions of E_0 and C_u can ensure reestablishment of the scale invariance.

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